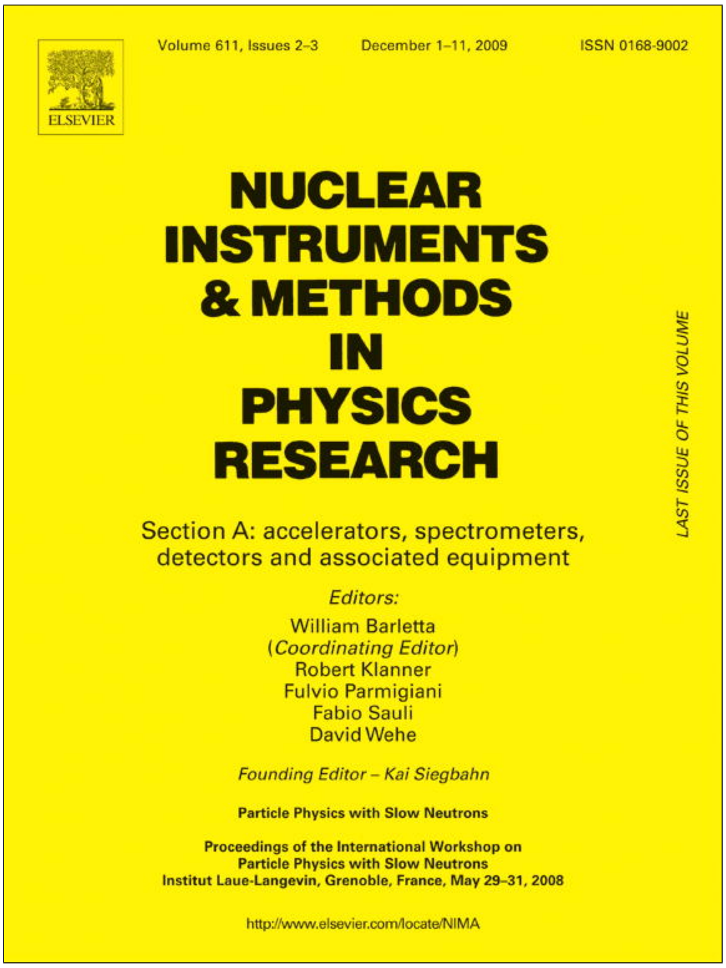
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Methods of observation of the centrifugal quantum states of neutrons

R. Cubitt [a], V.V. Nesvizhevsky [a,*], A.K. Petukhov [a], A.Yu. Voronin [b], G. Pignol [c], K.V. Protasov [c], P. Gurshijants [d]

[a] Institut Laue-Langevin (ILL), 6 rue Jules Horowitz, F-38042 Grenoble, France
[b] P.N. Lebedev Physical Institute, 53 Leninsky Prospekt, 119991 Moscow, Russia
[c] Laboratoire de Physique Subatomique et de Cosmologie (LPSC), IN2P3-CNRS, UJF, 53, Avenue des Martyrs, F-38026 Grenoble, France
[d] Institute of Solid State Physics (ISSP), Institutskaya Street 2, 142432 Chernogolovka, Moscow Region, Russia

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ABSTRACT

We propose methods for observation of the quasi-stationary states of neutrons, localized near a curved mirror surface. The bounding effective well is formed by the centrifugal potential and the mirror's optical potential. This phenomenon is an example of an exactly solvable "quantum bouncer" problem that can be studied experimentally. It could provide a new tool for studying fundamental neutron–matter interactions, neutron quantum optics and surface physics effects. The feasibility of observation of such quantum states has been proven in first experiments.

1. Introduction

The *centrifugal states* of neutrons are a quantum analog of the so-called whispering gallery wave, the phenomenon which in brief consists of the wave localization near a curved reflecting surface. It is known in acoustics since ancient times and was explained by Lord Rayleigh in his "Theory of Sound" [1,2]. The whispering gallery waves phenomenon in optics is an object of growing interest during the last decade [3,4]. In the following we will be interested in the matter–wave aspect of the whispering gallery wave phenomenon, namely the large-angle neutron scattering on a curved mirror [5]. Such a scattering can be explained in terms of the states of a quantum particle above a mirror in a linear potential—the so-called *quantum bouncer* [6]. The neutron quantum motion in the Earth's gravitational field above a flat mirror is another example of such a quantum bouncer, which was observed recently [7]. We will show that the centrifugal quantum bouncer and the gravitational quantum bouncer have many common features. Centrifugal quantum states of neutrons in another experimental configuration were considered by Watson [8].

The phenomenon of centrifugal neutron states consists in the localization of cold neutrons near a curved mirror surface due to the superposition of the effective centrifugal potential and the optical potential U_0 of the mirror (see Figs. 1 and 2). It plays the dominant role in the scattering of neutrons to large angles compared to the critical angle. Measurement of the gravitationally bound and the centrifugal quantum states of neutrons can be considered as a test of the equivalence principle for a quantum particle [9–13]. Both problems (the gravitational and the centrifugal one) provide an experimental laboratory for studying neutron quantum optics phenomena, quantum revivals and localization [14–21]. Evident advantages of using cold neutrons compared to ultracold neutrons include much higher statistics attainable, broad accessibility of cold neutron beams as well as a crucial reduction of many false effects compared to the experiments with the gravitationally bound quantum states of neutrons due to approximately 10^5 times higher energies of the quantum states involved. This phenomenon could provide a new tool for studying fundamental neutron–matter interactions (in analogy to Refs. [22–25]) as well as surface physics effects.

2. Principle of observation

If the neutron energy is much larger than the optical potential U_0 of a curved mirror most neutrons are scattered to small angles. However some neutrons can be captured into long-living centrifugal quasi-stationary states localized near the curved mirror surface and thus could be detected at large deflection angles. The curved mirror surface plays the role of a wave-guide and the centrifugal states play the role of radial modes within. The spectral dependence of the transmission probability and the neutron angular distribution is determined by the existence of the centrifugal states in such a system. The effective acceleration near the curved mirror surface can be approximated as $a = v^2/R$, where v is the neutron velocity and R is the mirror's radius of curvature. The classical radial motion of the neutron in this well is limited at

* Corresponding author.
E-mail addresses: nesvizhevsky@ill.eu (V.V. Nesvizhevsky), dr.a.voronin@gmail.com (A.Yu. Voronin).

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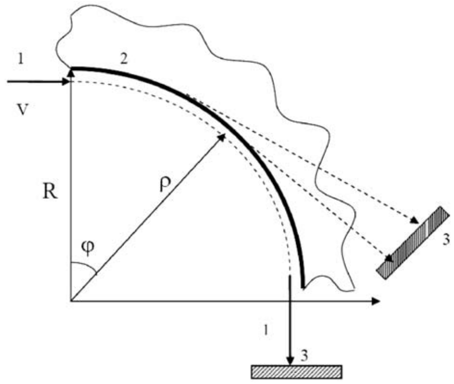

Fig. 1. Scheme of the neutron centrifugal experiment [5]. 1—classical trajectories of incoming and outgoing neutrons, 2—cylindrical mirror, and 3—detectors for deflected and for tunneling neutrons.

the height $z_{class} = RU_0/mv^2$ above the surface. The condition for the existence of a quasi-stationary state can be estimated from the Heisenberg relation

$$z_{class}\sqrt{2mU_0} > 2\pi\hbar \quad \text{or} \quad \frac{v^2}{R} < \frac{1}{2\pi\hbar}\sqrt{\frac{2}{m}}U_0^{3/2}. \tag{1}$$

It is natural to chose the neutron velocity v within the range of maximum intensity of standard neutron sources (reactor or spallation sources) around 10^3 m/s. The shortest wavelength should not be smaller than the Bragg cut-off associated with the atomic structure in the mirror material. This means we can treat the material in terms of a uniform optical potential. For neutron velocity around 10^3 m/s, the radius R for which Eq. (1) holds is a few centimeters.

When varying the velocity v, we change the width of the triangular barrier in Fig. 2 (i.e. the size of the centrifugal trap), changing also the number of quasi-stationary states that can propagate along the mirror. One thus expects a step-like behavior of the transmitted flux as a function of the velocity, using the optical potential of the mirror as a "filter" for the quantum states. Analogous step-wise dependence of the total neutron flux as a function of the slit size was observed in case of the gravitationally bound quantum states [7]. An alternative method for studying the centrifugal quantum states consists in measuring the velocity distribution in the quantum states using a position-sensitive neutron detector, placed at some distance from the curved mirror. Such a method was used as well in the experimental studies of the gravitationally bound quantum states of neutrons [26].

In contrast to the gravitationally bound quantum states neutrons tunnel deeply into the mirror through the triangular potential barrier shown in Fig. 2. The lifetime of deeply bound states is much longer than that for states with energies close to the barrier edge. This phenomenon has to be taken into account. Another essential difference is related to the effects of surface roughness: as the characteristic length scale of the centrifugal quantum states is much smaller (see below), the roughness effects are much larger; therefore the requirements for the cylindrical mirror surface are even more severe than those for flat mirrors in the gravitational experiments.

The detailed description of the theory of neutron scattering by a curved mirror can be found in Ref. [5]. The equation describing the neutron states localized near the curved mirror surface is obtained by expanding the centrifugal energy in the vicinity of the mirror surface in a linear approximation:

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + U_0\Theta(z) - \frac{mv^2}{R}z - \varepsilon_n\right]\chi_n(z) = 0. \tag{2}$$

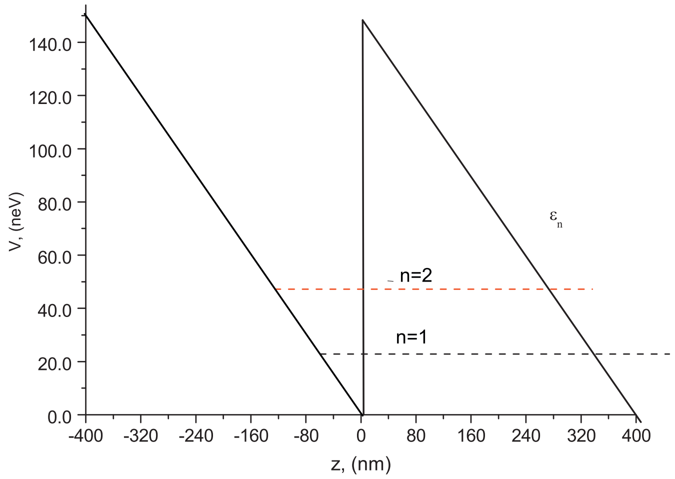

Fig. 2. Sketch of the effective potential in the vicinity of the mirror surface [5]. The potential step at $z = 0$ is equal to the mirror's optical potential $U_0 = 150$ neV. The potential slope at $z \neq 0$ is governed by the centrifugal effective acceleration $a = v^2/R$.

Here z is the deviation of the neutron coordinate from the mirror surface. The quantum states correspond to the solution of Eq. (2) with the outgoing wave boundary condition:

if $z > 0$

$$\tilde{\chi}_n(z) \sim \mathrm{Bi}\left(\frac{U_0 - \varepsilon_n}{\varepsilon_0} - \frac{z}{l_0}\right) + i\,\mathrm{Ai}\left(\frac{U_0 - \varepsilon_n}{\varepsilon_0} - \frac{z}{l_0}\right)$$

if $z \leq 0$

$$\tilde{\chi}_n(z) \sim \mathrm{Ai}\left(-\frac{z}{l_0} - \frac{\varepsilon_n}{\varepsilon_0}\right)$$

where Ai and Bi are the Airy functions and

$$l_0 = (\hbar^2 R/(2m^2v^2))^{1/3} \tag{3}$$

$$\varepsilon_0 = (\hbar^2 m v^4/(2R^2))^{1/3}. \tag{4}$$

Here l_0 and ε_0 are the characteristic quantum distance and the characteristic quantum energy scale respectively, defined by the solution of the Schrödinger equation. For the typical experimental setup parameters $U_0 = 150$ neV, $v = 1000$ m/s and $R = 2.5$ cm these scales are $l_0 = 0.04\,\mu$m, $\varepsilon_0 = 15.3$ neV, and $U_0/\varepsilon_0 \simeq 10$.

One can show [5] that the neutron flux parallel to the mirror surface integrated over the band $\rho_2 - \rho_1 = h \ll R$ is determined by the following expression:

$$F(\varphi) \approx \frac{v}{R}\sum_n |C_n|^2 \exp\left(-\frac{\Gamma_n \varphi R}{\hbar v}\right). \tag{5}$$

Here n indicates the quasi-stationary state number, $|C_n|^2$ its initial population, and Γ_n its width. The semiclassical expression for the width of the quasi-stationary state with energy λ_n is

$$\Gamma_n \simeq 4\frac{\varepsilon_0^2}{U_0}\sqrt{\frac{U_0 - \varepsilon_n}{\varepsilon_0}}\exp\left(-\frac{4}{3}\left(\frac{U_0 - \varepsilon_n}{\varepsilon_0}\right)^{3/2}\right). \tag{6}$$

The energy and the width of the quasi-stationary states depend strongly on the neutron velocity v. A small velocity results in a broad barrier; indeed, $U_0/\varepsilon_0 = U_0[(2R^2)/(\hbar^2 M v^4)]^{1/3}$ increases if v decreases. The widths of the quasi-stationary states decrease exponentially with decreasing v (see Eq. (6) and Fig. 3). The sharp increase in the lifetime of the quasi-stationary states with decreasing velocity can be exploited for the experimental observation of such states. Indeed, when the neutron velocity decreases the contribution of new quasi-stationary states increases rapidly. This results in a step-like dependence of the deflected neutron flux as a function of v. In Fig. 4 the calculated flux of deflected neutrons is shown as a function of the neutron velocity. Under the assumptions made above the problem of deflection of cold neutrons ($v \sim 10^3$ m/s) by the curved mirror is analogous to the problem of the passage of ultracold neutrons through the slit between a horizontal mirror and an absorber in the presence of the Earth's gravitational field, studied in detail in Refs. [27–34]. In the case of neutron motion along a curved mirror surface the initial velocity variation results in changing the spatial dimension of the effective well, which bounds the neutron near the surface, ensuring the "scanning" of the quasi-stationary states.

3. Experimental tests

In order to test the feasibility of observing the centrifugal quantum states, a series of test experiments were carried out at the cold reflectometer D17 [35] of the Institut Laue-Langevin (ILL).

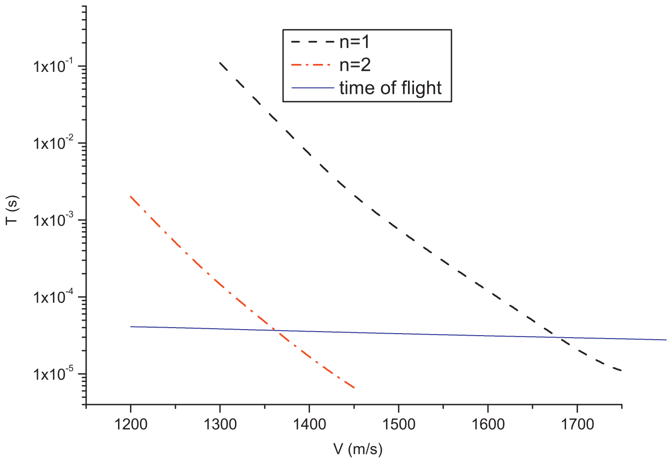

Fig. 3. Expected lifetimes of the two lowest neutron centrifugal quasi-stationary states as a function of the neutron velocity [5]. The mirror's radius of curvature is $R = 2.5$ cm, its length 5 cm, and its optical potential $U_0 = 150$ neV. The nearly horizontal solid line indicates the time-of-flight along the curved mirror.

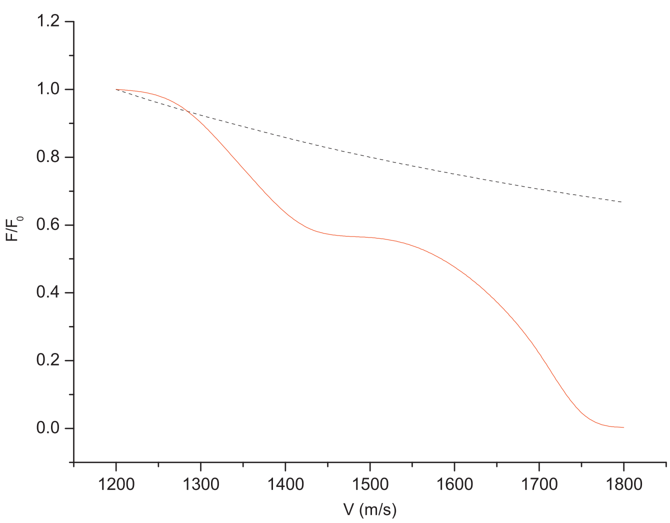

Fig. 4. Expected flux of neutrons (continuous line) deflected by the curved mirror, as a function of the neutron velocity, normalized to the calculated flux F_0 at $v = 1200$ m/s. The mirror's radius of curvature is $R = 2.5$ cm, its length $L_{mirr} = 5$ cm and its optical potential $U_0 = 150$ neV. The dashed line shows the classical expectation (normalized to the calculated classical flux at $v = 1200$ m/s).

The instrument was used in its time-of-flight configuration; the tested mirrors were installed in the sample position, and the spatial-resolving detector served to measure deviation angle and neutron wavelength (the latter by time-of-flight). Some preliminary measurements were also carried out with a time-of-flight setup at the cold neutron beam facility PF1B [36] of the ILL.

Two main questions were addressed: the preparation of a mirror with a surface quality sufficiently high to provide long lifetimes of the quantum states (surface roughness lower than $\sim$1 nm, waviness significantly lower than 10^{-3} rad, small surface impurities) and the suppression of background to obtain a sufficient signal-to-background ratio.

The latter is especially difficult because of the extremely low ratio of deflected to incoming neutron fluxes. This ratio was typically as low as 10^{-9}–10^{-5} even for the narrowest collimation of the incoming neutron beam. In a classical approximation, the width of the deviated neutron beam is equal to $z_{class} \sim 10^{-7}$ m, while the incoming beam width is $> 10^{-4}$ m. When calculating the overlap of a classical incoming neutron plane wave with the wavefunctions of neutrons in the centrifugal quantum states, one gets additional significant intensity losses depending on the neutron wavelength and the initial angular distribution. Finally, many neutrons are lost because of their finite lifetimes in the quantum states (tunneling through the triangle barrier and non-specular reflection at surface roughness, waviness and impurities). On the other hand, all other (then scattered) neutrons can produce background in the large position-sensitive detector needed for this experiment, because of their scattering in mirror material, in windows, and in air. In order to suppress scattering in the mirror material we used crystals (sapphire, silicon) for manufacturing the mirrors. In single crystals, also the neutrons tunneling through the triangle barrier into the mirror material move on well-defined trajectories and can thus be used for data analysis. Besides, we removed windows from the mirror vicinity, and for some measurements the mirrors were installed in vacuum. Neutron collimators were always installed to absorb scattered neutrons arriving from all directions except for direct view between the mirror exit and the position-sensitive detector. Finally, a high signal/background ratio required large intensity of deflected neutrons, which is long lifetimes of neutrons in the quantum states.

For producing the mirror, we identified two strategies: bending of extra-thin well-polished crystal plates and cutting of a sector of a thick-wall polished tube.

First, we studied thin plates of well-polished single-crystal sapphire and silicon. The typical surface roughness was equal to 0.4–0.8 nm and could be significantly improved if needed. The surface flatness of non-bended plates was much better than 10^{-3}. Then such high-surface-quality plates were bended to a cylindrical shape with minimum radius. In order to provide the calculated cut-off wavelength in the deflected neutron beam close to the maximum intensity in the incoming neutron beam (4–5 Å), the cylinder radius had to be as small as 2–4 cm. In order to get such small radius of curvature we had to use plates with minimum thickness. However, the smallest plate thickness is limited by the mechanical properties to 100–200 μm; thinner plates are broken when polishing. Unfortunately, such plates could not be curved to a cylinder with a radius < 10 cm; the plates are broken when curved any further. Moreover, the shape of the plates curved to minimum radius is poorly defined as confirmed by both laser and neutron measurements (the local radius of curvature varies along the surface at a level of $\sim 10^{-2}$, also the macroscopic shape of the plate is poorly defined). Due to the small amount of material exposed to the neutron beam, such experiments provided very low backgrounds. To summarize: the first strategy enabled us to meet all mentioned requirements concerning the surface quality of non-bended plates and permits low backgrounds, but it did not provide a perfect cylindrical shape and sufficiently small radius of the mirrors.

Second, we studied sectors of thick-wall polished sapphire tubes. A result obtained using this strategy is shown in Fig. 5. Evidently, the background of scattered neutrons was sufficiently low (however not as good as that for thin plates). The central spot corresponds to deflected neutrons, coming out of the exit mirror edge. The tail on the left side of the spot corresponds to neutrons tunneling from the centrifugal quantum states into the mirror bulk through the triangle barrier. So, large-angle deflection of neutrons at a curved mirror has been observed. A wavelength cut-off in the deflected neutron flux seems to have been observed as well. However, the cut-off wavelength value was larger than expected. This is most probably due to defects at the mirror surface. This conclusion is supported by detailed studies of the mirror surface by atomic force microscopy. Although the average roughness amplitude was nearly acceptable ($\sim$2 nm), the distribution of amplitudes had a specific shape. An enhanced fraction of surface waves with very large amplitude (10–50 nm) and significant aperture ($(1–5)\times10^{-3}$) dominated the probability of neutron scattering, as confirmed by numerical estimations. The specific shapes of surface defects corresponded to intense neutron scattering to neighbor quantum states, thus keeping the total intensity of deflected neutrons relatively high but washing out any quantum structure of the deflected neutron flux and ruling out any reliable quantitative analysis of the data. Such large-scale defects originated from the hardness of the material used and from insufficient time of polishing. In order to get rid of such defects and provide an average roughness of <1 nm, one could use softer material (silicon), longer polishing time and better control of surface parameters during polishing. Besides, one should note that the optical potential of silicon is twice smaller than that of sapphire; therefore requirements for residual roughness are significantly lower. To summarize: the experiment has demonstrated the feasibility to observe the centrifugal quantum states of neutrons. Results of an experiment using a curved mirror free of the listed defects are being analyzed, and they will be presented later on.

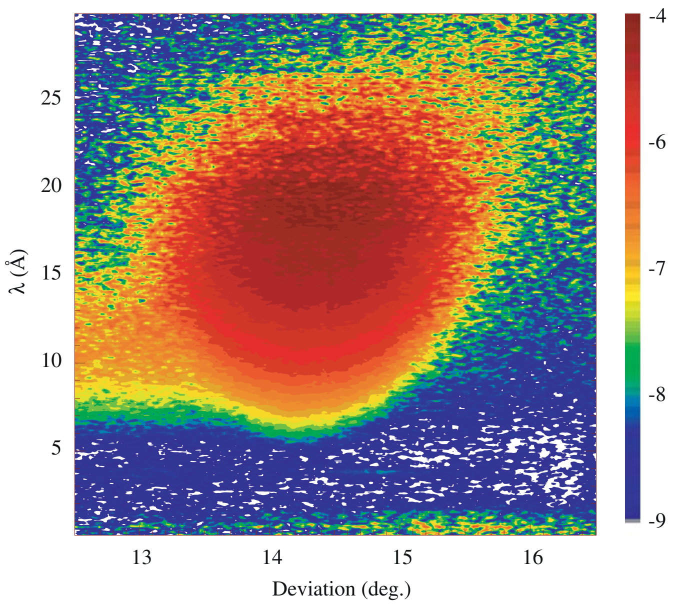

Fig. 5. Measured number of deflected neutrons as a function of neutron wavelength and deviation angle, normalized to the total incoming neutron number for each wavelength bin. The geometry of the experiment is shown in Fig. 1. The radius of the sapphire mirror is 25 mm, its height 20 mm, and its angular size $14.5\pm0.5^{\circ}$. The color scale is decadic logarithmic, the bin size $0.31^{\circ}\times0.12$ Å. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

4. Conclusions

We have proposed methods for observation of the quasi-stationary states of neutrons localized near a curved mirror surface. The effective bounding well is formed by a superposition of the centrifugal potential and the mirror's optical potential. Reduction of the initial neutron velocity results in an increase in the spatial size of such a centrifugal trap which, in its turn, results in the appearance of quasi-stationary states in the spectrum of the system. This could be observed as a step-like dependence of the deflected neutron flux. Simultaneously the centrifugal quantum states could manifest in a specific angular dependence of the transmitted neutron flux, as well as in the flux of the neutrons tunneling through the triangular potential. Such neutron states could provide a promising tool for studying different types of neutron–matter interactions with the characteristic range of a few tens of nm. The feasibility of observation of such quantum states has been proven experimentally.

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