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On an experiment related to the observation of quantum states of a neutron in a gravitational field.

Abstract.

A neutron can occupy quantum stationary states when it is trapped between the Earth's gravitational field on one side and the Fermi potential of a bottom mirror on the other side. This was pointed out for the first time by V.I.Luschikov and A.I.Frank [1,2]. They suggested a principal scheme of an experiment and discussed its main problems. Here a relatively easy method of observation of these quantum states is proposed. An enlarged image of a space variation in neutron density along the vertical axis near the mirror surface can be obtained using an one-mirror optical scheme. Good statistics in this experiment is provided by the use of a position sensitive UCN detector which allows one to measure simultaneously the position probability density distribution in every range of interest. The existence of these quantum energy levels causes strong variation in a neutron density both for separate energy levels and for a mixture of low energy states. This fact simplifies significantly the technical requirements for an experiment and it increases the counting rate. The resulting image probes details of the interaction of neutrons with a surface, which provides a tool for investigating this interaction.

1. Motivations of the experiment.

For one, this is a beautiful physical phenomenon. A range of parameters of this system (an elementary particle in a field) is unusual: the energy separation of neighbour levels is about 10^{-12} eV, a neutron gains this energy falling down a distance of 0.01mm(!), which is in the macroscopic range.

The proposed experiment is a relatively easy method of observing quantum stationary states of an object in a gravitational field. This would be for the first time that such quantum stationary states are observed. Measuring gravitational levels puts constraints on the type of experiment. Quantum effects are negligible for macroscopic masses, therefore elementary particles can be used only. Charged elementary particles cannot be used because they undergo strong electromagnetic interaction. Electrical neutral atoms are light which makes them in principle attractive. So, one needs an electrical neutral elementary particle with a long life time *i.e.* a neutron (or, perhaps, an atom). The energy range of UCNs (ultra cold neutrons) allows a reasonable installation size, and the neutron fluxes available ensure reasonable counting statistics [3,4].

Is the existence of these quantum states evident? This is not obvious. If the life time of a neutron in a stationary state is small (providing, the condition from the uncertainty principal, ΔE equals to the energy separation of neighbour levels) than neighbouring gravitational levels will overlap. This can happen for example due to anomalous cooling/heating which is currently under investigation by A.Steyerl [5]. And, in any case, this anomalous cooling/heating would have an important influence to the process which we are interested in.

The measurement of widths and neutron populations at different levels is a sensitive tool to investigate mirror reflections. It also provides useful information about reflecting surfaces (the horizontal mirror - fig.5) and absorbing/scattering surfaces (the absorber/scatterer over the horizontal mirror - fig.5). If the quantum stationary states of a neutron in a gravitational field are observed experimentally, numerous experiments with resonance transitions of a neutron from one level to another and measurements using a magnetic field could be considered.

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2. Mathematical description of stationary states of a neutron in a gravitational field.

2.1 General solution.

Let's follow briefly the standard mathematical description of a particle above an infinite horizontal mirror in the Earth's gravitational field. V.I.Luschikov and A.I.Frank applied the general solution of this system (see for example [6]) to the case of a neutron [1,2].

The potential energy of a neutron equals :

$$U(z) = \begin{cases} \infty, z \le 0 \\ mgz, z \ge 0 \end{cases} \tag{1}$$

A neutron undergoes total reflection from a mirror surface, if the energy of its vertical motion is lower than the Fermi potential. This is the case for all materials with positive Fermi potential. From another side it is trapped by the Earth's gravitational field. The potential is uniform in the horizontal plane. This means that there are quantum stationary states which can be described by means of the one dimensional Schrödinger equation for the vertical component:

$$-\frac{h^2}{2m}\frac{d^2\psi}{dz^2} + mgz\psi = E\psi \tag{2}$$

The general solution of this equation is:

$\psi = A\phi(\xi)$, where $\phi(\xi)$ - is the Airy function and A - is a normalization coefficient. (3)

Energy levels correspond to roots of the following equation:

$$\phi\left(-\frac{\sqrt[3]{2}}{\sqrt[3]{mg^2h^2}}E\right) = 0 \tag{4}$$

Stationary state wave functions of corresponding energy levels $\psi_n(z)$ are given by:

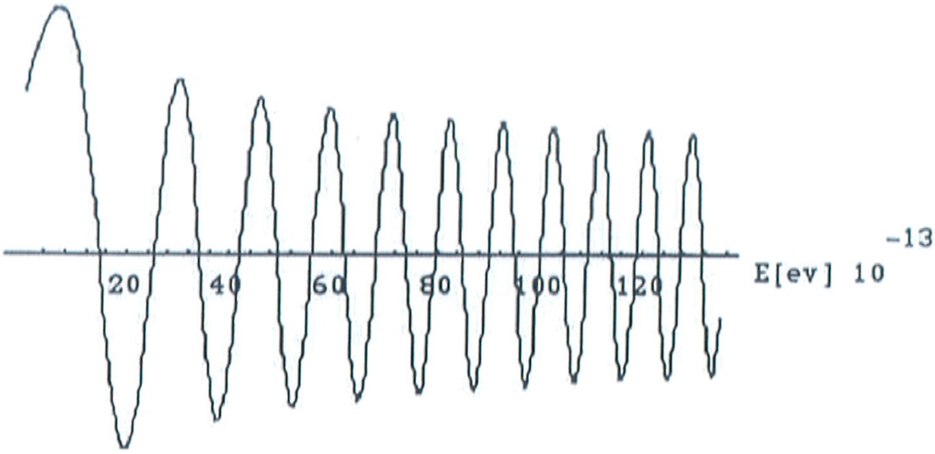
$$\psi_n(z) = A_n(z)\phi\left(\frac{z}{a} - \alpha_n\right),$$

where $\phi(\frac{z}{a} - \alpha_n)$ - is the Airy function, $a = \sqrt[3]{\frac{h^2}{2m^2g}}$,

α_n - corresponding roots of the Airy function, and $A_n = \dfrac{1}{\sqrt{a\int_{-\alpha_n}^{\infty}\Phi(\zeta)^2 d\zeta}}$ (5)

2.2 Energy levels.

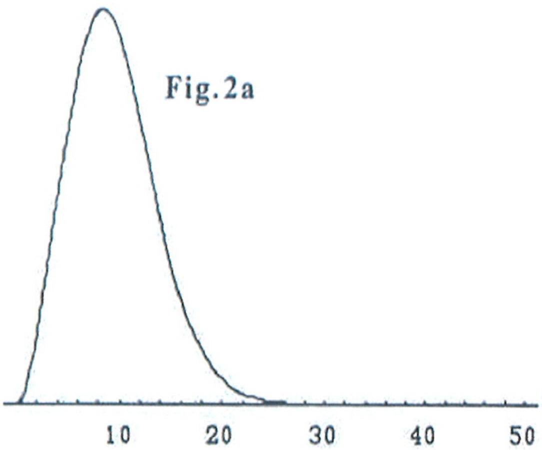
Fig. 1 The roots of ϕ-function (eq.4) correspond to the values of the energy levels. Energy units are chosen is a special way: 10^{-13} eV is the energy that UCN gains in the Earth's gravitational field falling down a distance of about 1μ. This presentation is convenient to estimate the space scale involved.



The energy of the lowest level is $14.4 \cdot 10^{-13}$eV, the next four levels lie at $25.3 \cdot 10^{-13}$eV, $34.2 \cdot 10^{-13}$eV, $42.1 \cdot 10^{-13}$eV and $49.3 \cdot 10^{-13}$eV. The low energy levels are well resolved, the energy separation of higher levels is smaller.

2.3. The probability density of the neutron position.

The neutron position probability density along the vertical axis is proportional to $\psi_n^2(z)$ (eq.5). These dependencies for 1st, 2nd, 3d and 4th quantum levels are shown in fig.2a,2b,2c,2d respectively:



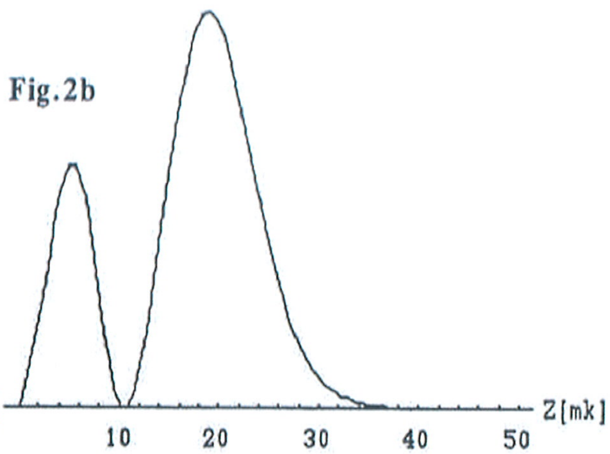


Fig.2c

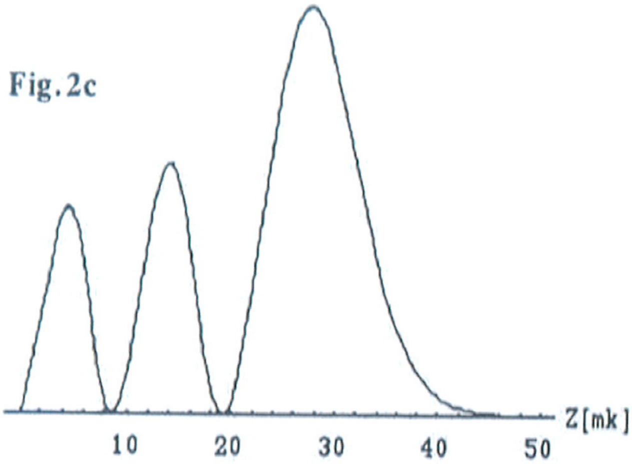
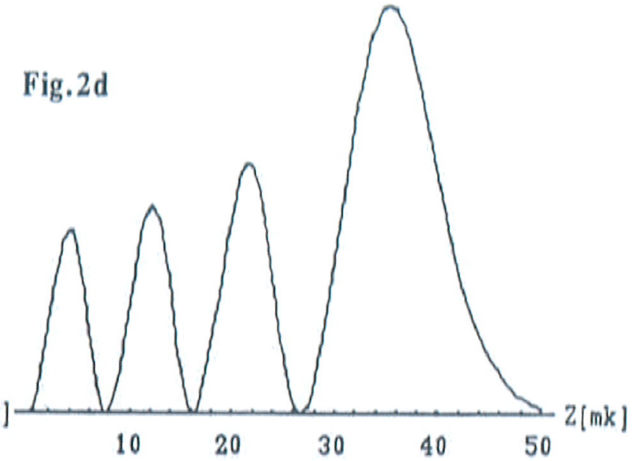


Fig.2d



The position probability density function $\psi_n^{\,2}(z)$ has (n) maximums and (n-1) minimums where the probability is zero as in any case of an ordinary standing wave. Fig.2 shows that the differences between the maximums and minimums are well distinguishable.

In order to estimate the effect that has on $\psi^2(z)$ mixing of different quantum states one needs to use the neutron population at different energy levels. In classical approximation. the population (flux) is proportional to $E^{1.5}$:

$$\psi^2(z) = \frac{1}{N}\frac{\sum_{n=1}^{N}\psi_n^{\,2}(z)(E_{n+1}^{\;1.5} - E_{n-1}^{\;1.5})}{\sum_{n=1}^{N}(E_{n+1}^{\;1.5} - E_{n-1}^{\;1.5})} \qquad (6)$$

Once the phenomenon is established experimentally a full quantum mechanical treatment of the system can be done. The resulting position density distributions for mixing of 2, 4 or 8 levels are shown in fig.3a,3b,3c respectively. The variation of neutron density is still large even for a mixture of several levels. Moreover, it can be strongly enhanced near the mirror surface if the lowest energy level is cut out. Corresponding distributions are shown in fig.4a,4b,4c respectively. The cut of the first level can be done by means of a barrier/absorber on the mirror surface (see par. 4.3).

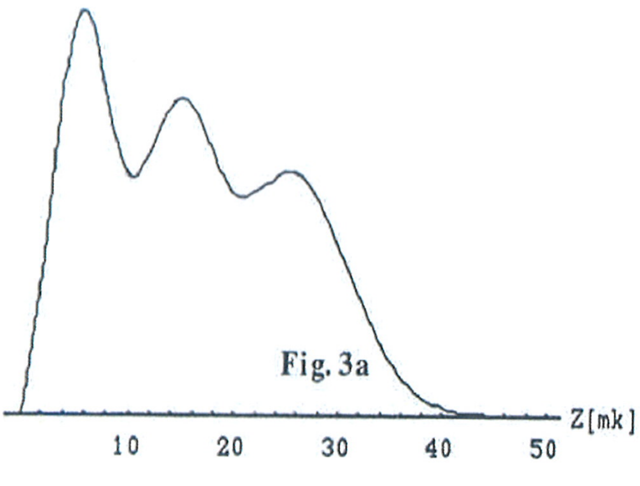


Fig. 3a

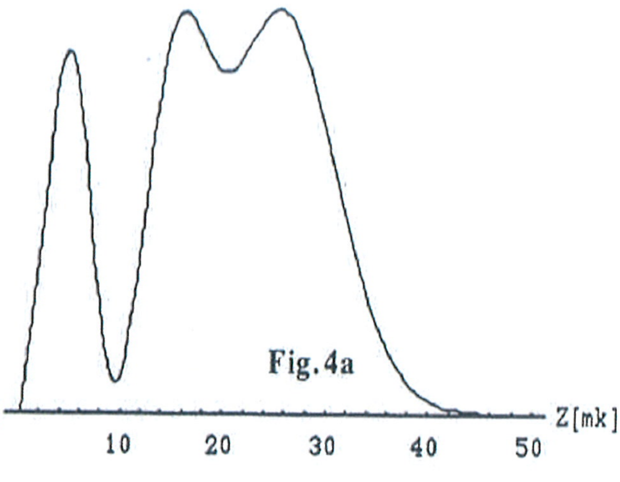


Fig. 4a

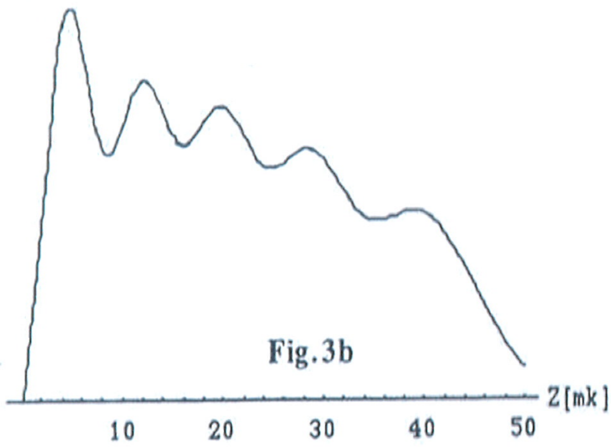


Fig.3b

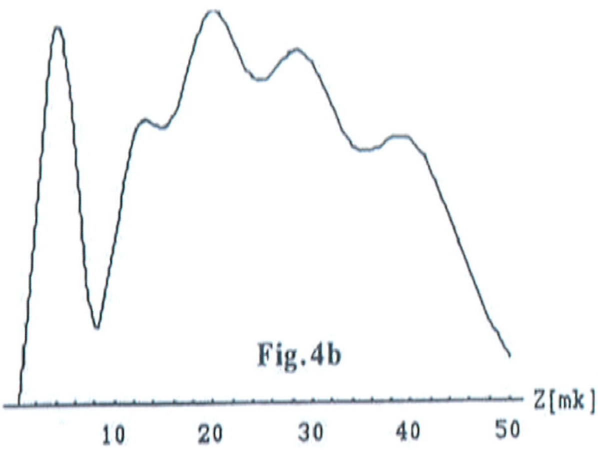


Fig.4b

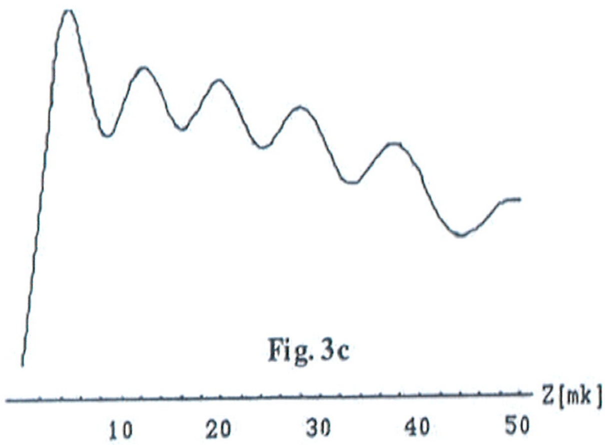


Fig. 3c

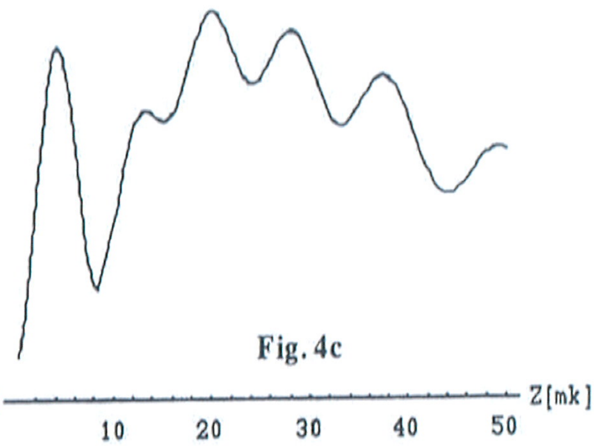


Fig. 4c

3. Scheme of the experiment.

An energy resolution of ΔE=10^{-13}eV has a corresponding time interval from the "uncertainty principal" that equals $\Delta T \approx \dfrac{h}{\Delta E} \approx 6.6 \cdot 10^{-3} s$. This corresponds to a distance of 3.3cm for velocity $5\dfrac{m}{s}$. On other hand, in classical approximation, the time interval between two consequent collisions with the mirror bottom equals $\Delta T_2 = 2\sqrt{\dfrac{2h_1}{g}} \approx 3.4 \cdot 10^{-3}$s for a neutron at the 1st level and $6.3 \cdot 10^{-3}$s at the 5th level. Hence, very few collisions are enough to resolve the energy levels.

So, if one has a neutron for about 10^{-2}s above a horizontal mirror than there is the possibility to observe its quantum stationary states in the Earth's gravitational field. This defines the choice of UCN for this measurement baring in mind that the installation size should not be too large. On other hand 10^{-2}s is a short time, which means that one do not need to store neutrons but just to use a "flow through" mode. This simplifies the measurement considerably.

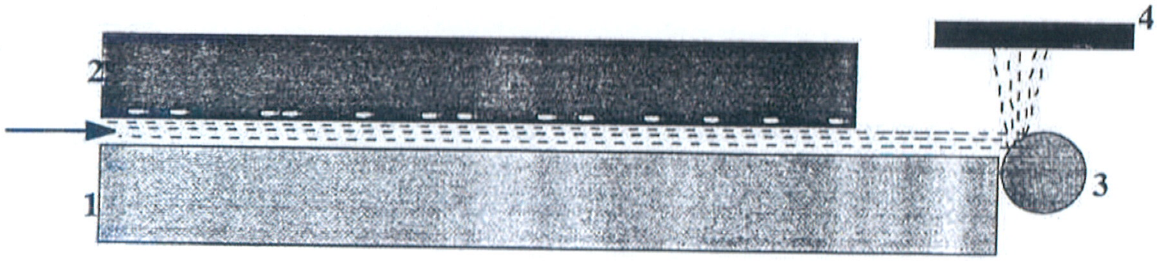
In order to vary the position distribution in controllable way one should be able to cut out the neutrons with "high" energies. This cut can be done, for example, at a height of 50μ which corresponds to the 5th energy level (corresponding range of heights is shown in fig.3 and fig.4). Unfortunately the standard method of cutting an UCN spectrum using an absorber at the corresponding height does not work because the absorber needs an extremely low Fermi potential. However in practice it is never lower than the energy of vertical motion of a neutron at the first energy level. If so than a neutron is reflected. If this reflection is specula one than there is no cut. Therefore a slight modification of this method is required: high probability of nonspecula reflection at the absorber's surface needs to be created.

To be precise, one should mention also that the solution to the Schrödinger equation (eq.2) does not correspond to experimental conditions because the absorber with non zero Fermi potential modifies wave functions and energy levels. However this influence is negligible for energy levels lower than the absorber/scatterer height. On other hand neutrons at higher energy levels are cut (in the ideal case) by this absorber/scatterer. So the final density distribution is the same.

The horizontal component ($\approx 500\frac{cm}{s}$) of the velocity of the neutron arriving at the cylindrical mirror is much larger then the vertical component ($< 5\frac{cm}{s}$). The neutron density in the beam depends on its distance to the horizontal mirror. Therefore in the vertical slide of the beam one can find several "jets" of neutrons. These jets are homogenous in the horizontal plane.

Unfortunately they can not be observed directly by means of a position sensitive neutron detector, because the required position resolution of 1μ is not available. Besides, without a horizontal mirror under the beam. the jets overlap after a few tens of mm. Both these problems can be overcome by using a cylindrical mirror placed close to the bottom mirror plate. Incoming neutrons will be spread out by the cylindrical mirror enhancing the position resolution by a factor of 50. A standard scintillating detector with position resolution about 50μ can than be used. The scheme of the set-up is shown in fig.5:

Fig.5 Principal scheme of the setup (for clarity the cylinder diameter has been enlarged) : 1) horizontal glass mirror, 2) absorber/scatterer, 3) cylindrical mirror, 4) position sensitive neutron detector for UCN.



The cylindrical mirror does not affect the neutron distribution in the plane parallel to its axis. The amplification in other plane depends on the radius of the cylindrical mirror and on the distance to the detector. The non linearity of the image thus obtained is not important for a first observation of stationary states but it can be easily improved (if necessary) by more complicated optical scheme in the future. There is no constraint from the uncertainty principal which does not allow enlarge the image, because the horizontal component of neutron velocity is large (few meters per second). So the reflection from the cylindrical mirror can be treated using classic optics.

4. Discussion of possible problems.

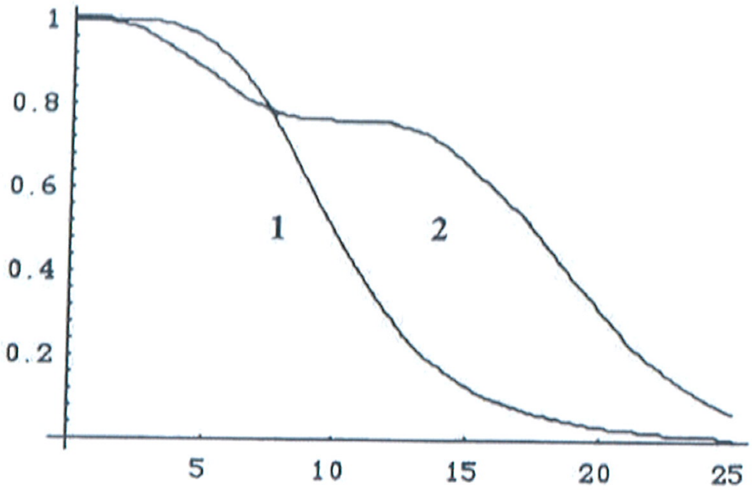
There is a complicated but evident problem of precise and careful manufacturing and adjusting the optical elements in the experiment. However, the required position resolution of 1μ can be obtained using standard optical devices.

Also vibrations of the horizontal mirror can destroy the gravitational levels. Due to interaction with moving wall UCNs gain (or loss) a velocity of 2 times the velocity of the wall. In our case of measuring 5 energy levels vibrations exceeding $1\frac{mm}{s}$ would result in possible jumps from one gravitational level to another. This would be even worse for neutrons being reflected several times along the path in the system. As result a maximum velocity of the mirror $0.1\frac{mm}{s}$ is required.

The resulting neutron position distribution is sensitive to the efficiency of the absorber/scatterer, which depends on the interaction of the UCN with the surface. However, the need to know this efficiency is reduced if the vertical velocity spectrum is well cut due to several collisions with the absorber/scatterer. The number of collisions depends on the length of the absorber/scatterer. The length of 10-15cm is sufficient, because, in classical approximation, a neutron at the 5th energy level travels only $3.2cm$ between two consecutive collisions. Also even in a not perfect cut in the velocity spectrum there will still be a different but important variation of the position probability density near the surface of the horizontal mirror.

In order to enhance the variation in position probability density it is important to be able to vary the experimental conditions in such a way that UCN at the lowest level are cut out without cutting in the rest of the spectrum. This can be done using a barrier/absorber on the surface of the horizontal mirror (fig.6):

Fig.6 Neutron fluxes vs the barrier/absorber height: 1) relative neutron flux at the 1st level normalized to one at the 2nd level, 2) absolute neutron flux at the 2nd level.



The neutron fluxes in fig.6 were calculated assuming a loss of flux is proportional to the integral of the neutron density over the distance between the mirror surface and the barrier/absorber height. For a barrier/absorber height of 16μ one loses 40% of neutron flux at the 2nd energy level provided that the flux at the 1st energy level is suppressed by a factor of 10 times compared to the flux at the 2nd level.

The position of the cylindrical mirror relative to the glass is chosen in such way that the jets are reflected vertically. Then the gravitational aberrations are minimal. For a distance of few mms between the cylindrical mirror and the detector with $V_{\min} \approx 3.5\frac{m}{s}$ the corresponding aberration is negligible.

There is one constraint to the maximal diameter of the cylindrical mirror coming from the final angular divergence of UCNs in a jet. This condition can be written in the following way: $\frac{\sqrt{2}\cdot 2(\mu)}{r(\mu)} \approx \frac{V_{vert.}}{V_{horiz.}}$, this gives a maximal radius of 300μ for an energy resolution of 10^{-13}eV. From purely technical point of view a cylinder with this "big" diameter of about 0.6mm is easier to make than smaller cylinder. If we are interested in one or two lowest energy levels only and the neutrons with low horizontal component of velocity are cut out than this constraint is even softer.

5. Estimate of statistical accuracy.

What total counting rate can be achieved with a "pilot" version of the experiment? A UCN flux of about $0.05\frac{n}{cm^3(\frac{m}{s})^3}$ [3] can be obtained at the entrance slit. The maximal neutron flux at the detector can be estimated in the following way:

$$N[\frac{n}{s}] \approx 1.7 \cdot 0.05 \cdot \Delta h \cdot \Delta l \cdot \frac{D}{L} \cdot 2 \cdot \sqrt{2 \cdot g \cdot \Delta h} \cdot \frac{(V_{max}{}^3 - V_{min}{}^3)}{3} \tag{7}$$

with

Δh - the height of the slit between the horizontal mirror and the absorber/scatterer (0.005cm),

Δl - the width of the detector (2.5cm),

D - the width (10cm) and L - the length (15cm) of the horizontal mirror,

$V_{min} \approx 3.5\frac{m}{s}, V_{max} \approx 6\frac{m}{s}$.

For the above dimensions and installation parameters the maximal counting rate is about $0.25\frac{n}{s}$. Neutrons lost before arriving at the detector and final detector efficiency decrease this value. Nevertheless even with a decrease of a factor 10 the counting rate ($0.025\frac{n}{s}$) is higher than the background which can be achieved with position sensitive UCN detector. To measure the height's density distribution with position resolution of 1μ (50 points) with a statistical accuracy of 5% in each point would take 9 days. There are ways of improving statistics: 1) using wider neutron spectrum if the cylindrical mirror with a Ni coated, 2) using a bigger detector, 3) using 2π-geometry. But even first estimation seems to be promising.

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